Simultaneous Linear Equations

Example:- Solve the system $(D \equiv \frac{d}{dt})$

$$(D^2-2)x - 3y = e^{2t} \quad \text{—— (1)}$$

$$(D^2+2)y + x = 0 \quad \text{—— (2)}$$

Find the particular solution if the initial conditions are $x = y = 1$, $Dx = Dy = 0$ when $t = 0$

Solution:→
operate on (1) with D^2 to obtain

$$D^4x - 2D^2x - 3D^2y = 4e^{2t} \quad \text{—— (3)}$$

From (1) $D^2x = e^{2t} + 2x + 3y$ & From (2) $D^2y = -x - 2y$
putting this value in (3), we have

$$D^4x - 2(e^{2t} + 2x + 3y) - 3(-x-2y) = 4e^{2t}$$

$$\Rightarrow D^4x - x = 6e^{2t}$$

$$\Rightarrow (D^4-1)x = 6e^{2t} \quad \text{—— (4)}$$

$$\therefore x = c_1e^t + c_2e^{-t} + c_3\cos t + \frac{2}{5}e^{2t} \quad \text{—— (5)}$$

~~& From (2)~~

And then using (2)

$$y = \frac{1}{3}\left[(D^2-2)x - e^{2t}\right]$$

$$= -\frac{1}{3}(c_1e^t + c_2e^{-t}) - (c_3\cos t + c_4\sin t) - \frac{1}{15}e^{2t} \quad \text{—— (6)}$$

For the particular solution

When $t=0$

$$x = c_1 + c_2 + c_3 + \frac{2}{5} = 1$$

$$y = -\frac{1}{3}(c_1+c_2) - c_3 - \frac{1}{15} = 1$$

$$Dx = c_1 - c_2 + c_4 + \frac{1}{5} = 0$$

$$Dy = -\frac{1}{3}(c_1 - c_2) - c_4 - \frac{2}{15} = 0$$

Then $c_1 = \frac{3}{4}$, $c_2 = \frac{7}{4}$, $c_3 = -\frac{19}{10}$, $c_4 = \frac{1}{5}$.

putting these values in (5) & (6) the required particular solution is obtained.

Ex-(2) Solve $\frac{dx}{dt} - 7x + y = 0$, $\frac{dy}{dt} - 2x - 5y = 0$

Solution:→ In the operator notation, the given system of equations becomes

$$(D-7)x + y = 0 \quad \text{—— (1)}$$

$$-2x + (D-5)y = 0 \quad \text{—— (2)}$$

$\therefore$ $-2\times$(1) + $(D-7)$(2), i,e;

$$-2(D-7)x - 2y = 0$$

subtracting

$$-2(D-7)x + (D-7)(D-5)y = 0$$

$$+ \qquad\qquad -$$

$$-(D-7)(D-5)y - 2y = 0$$

$$\Rightarrow \{(D-7)(D-5) + 2\}y = 0$$

$$\Rightarrow (D^2 - 12D + 37)y = 0$$

$\because$ A.E $\equiv D^2 - 12D + 37 = 0 \quad \Rightarrow D = 6 \pm i$

$$\therefore y = e^{6t}(c_1 \cos t + c_2 \sin t) \quad \text{—— (3)}$$

Again $(D-5)\times(1) + 1\times(2)$, we have

Subtracting

$$
\begin{array}{rcl}
(D-5)(D-7)x & + \cancel{(D-5)y} & = 0 \\
-\ 2x & + \cancel{(D-5)y} & = 0 \\
+ & - & \\
\hline
(D-5)(D-7)x + 2x & & = 0
\end{array}
$$

$$\Rightarrow (D^2 - 12D + 37)x = 0$$

$\therefore$ A.E $\equiv D^2 - 2D + 37 = 0 \quad \Rightarrow D = 6 \pm i$

$$\therefore x = e^{6t}(c_3 \cos t + c_4 \sin t) \quad \text{—— (4)}$$

Now if we take equations (3) & (4) as the solution of the given system, certain restrictions are required on the constants c_1, c_2, c_3, c_4

~~Example(2)~~ $\because \dfrac{dy}{dt} - 5y = 2x$

Substituting (3) & (4) in the above, we get

$$e^{6t}\{-c_1 \sin t + c_2 \cos t + 6(c_1 \cos t + c_2 \sin t)\}$$
$$- 5e^{6t}(c_1 \cos t + c_2 \sin t) = 2e^{6t}(c_3 \cos t + c_4 \sin t)$$

$$\Rightarrow (-c_1 + 6c_2 - 5c_2) = 2c_4$$

$$\Rightarrow c_2 + 6c_1 - 5c_1 = 2c_3$$

i.e; $c_2 - c_1 = 2c_4$ & $c_1 + c_2 = 2c_3$

$$\therefore x = \frac{1}{2}e^{6t}\{(c_1+c_2)\cos t + (c_2 - c_1)\sin t\} \quad \text{—— (5)}$$

Thus (3) & (5) are the solution of the given system of equations.